

## Performance study on solar assisted heat pump water heater using CO<sub>2</sub> in a transcritical cycle

M. Raisul Islam<sup>a</sup>, K. Sumathy<sup>a</sup>, J. Gong<sup>a</sup>, Samee Ullah Khan<sup>b</sup>

<sup>a</sup> Department of Mechanical Engineering

<sup>b</sup> NDSU – CIIT Green Computing and Communications Laboratory  
North Dakota State University, 58108-6050 Fargo ND (U.S.A)

Phone: 701-231-7139, fax: 701-231-8913, e-mail: [Sumathv.Krishnan@ndsu.edu](mailto:Sumathv.Krishnan@ndsu.edu)

**Abstract:** This study reports the numerical analyses of a CO<sub>2</sub> transcritical cycle on solar assisted heat pump water heating system in which evacuated tube U-pipe solar collector is utilized as an evaporator. This simulation model can predict operating characteristics for moderate meteorological conditions of North Dakota. The main purpose of this work is to simulate the overall performance of the heat pump cycle by varying compressor speed. From the analysis the coefficient of performance (COP) value is predicted from 1.5 – 2.8 with three different average solar insolation. The results showed that the performance of the system significantly influenced by the compressor speed and solar irradiation. This study also calculated the instantaneous collector efficiency and found in the range of 50 – 55%.

### Key words

Evacuated tube solar collector, solar assisted heat pump, R744 (CO<sub>2</sub>), transcritical thermodynamic cycle.

### 1. Introduction

A direct expansion solar assisted heat pump (DX-SAHP) system employs solar collector as an evaporator. A refrigerant that passes through the solar collector expands directly by the useful heat gain from the solar radiation and undergoes a phase transition phenomena from liquid to the vapor state. This useful heat gain by the refrigerant can be improved by maintaining the evaporator temperature slightly higher than the ambient conditions.

Higher the temperature difference between the evaporator and the surroundings result in higher heat losses reflecting in lower efficiency of the system. Hence, it is preferable to maintain the evaporator (collector) temperature at low or closer to ambient temperature in order to minimize heat losses from the collector. However, it is preferred to operate the evaporator in the temperature range of 0° – 10°C higher than the surroundings, because of the fact that, higher temperature effects higher heat gain [1]. Moreover, proper selection of the solar collector can further minimize the heat losses. Selective surface coatings along with the vacuum insulation between the two concentric tubes are some of the distinct design aspects of the evacuated tube solar collector over flat plate solar collector. Thus evacuated tube collector enhances higher useful heat gain along with minimal heat losses [2]. Hence, these types of collectors are currently widely used in the solar adverse regions for residential solar thermal applications. However, glass evacuated tubes cannot sustain high pressure system. Therefore, U-pipe inserted evacuated tubes are used to deal with a high pressure thermodynamic cycle in the solar heat pump systems.

The refrigerant characteristics also have a great impact on the performance of DX-SAHP system. However, many conventional refrigerants raise concern on their detrimental effects on the environmental issues.

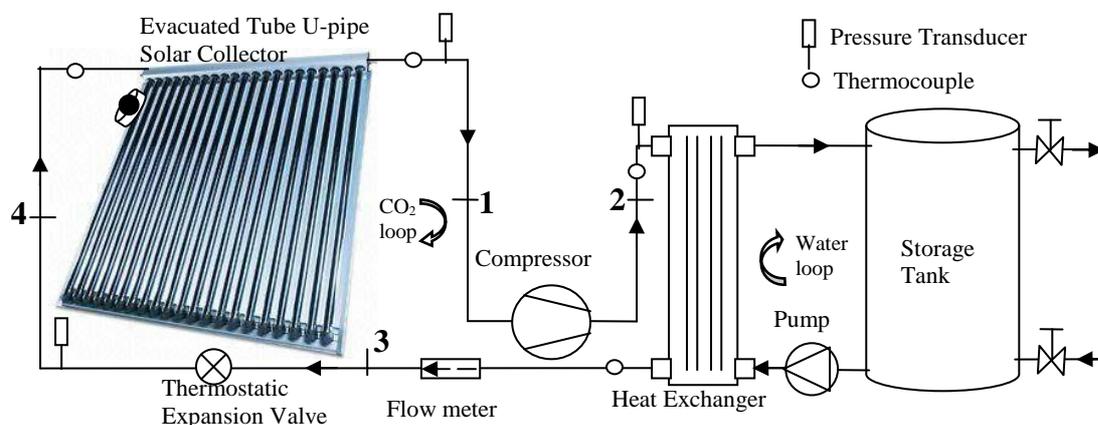


Fig. 1: Schematic diagram of a solar assisted direct-expansion (DX-SAHP) Water heating system.

## Nomenclature

$A$	Area
$c_p$	specific heat capacity (kJ/kg·K)
$C_{min}$	minimum heat capacity rate (kW/s·K)
$C_b$	bond conductance, W/(m.K)
$d$	diameter, m
$F'$	collector efficiency factor
$F$	fin efficiency
$I$	solar radiation (W/m <sup>2</sup> )
$h'$	heat transfer coefficient, W/(m.K)
$h$	specific enthalpy (kJ/kg)
$U_L$	overall loss coefficient (W/m <sup>2</sup> .K)
$U_t$	top loss coefficient (W/m <sup>2</sup> .K)
$U_e$	edge loss coefficient (W/m <sup>2</sup> .K)
$U_A$	convective heat transfer coefficient, (W/m <sup>2</sup> .K)
$\dot{m}$	mass flow rate (kg/s)
$\dot{m}_c$	compressor speed (rpm)
$N$	pressure (kPa)
$P$	heat transfer rate (kW)
$Q$	temperature (K)
$T$	swept volume of compressor (m <sup>3</sup> )
$V_s$	compressor power (kW)
$W_{com}$	circumferential distance between U-tubes, m
$W$	constant
$m$	

## Greek Letters

$\alpha$	absorptance of the collector
$\tau$	transmittance of the collector
$\epsilon$	counter-flow NTU effectiveness
$\eta$	efficiency, %
$\lambda$	conductivity of fin W/(m.K)
$\delta$	thickness of fin, m
$\rho$	density (kg/ m <sup>3</sup> )

## Subscripts

$a$	ambient
$coll$	collector
$dis$	discharge
$suc$	suction
$s$	isentropic
$f$	fluid
$i$	inlet
$o$	outlet
$ref$	refrigerant
$g$	glass
$w$	water
$u$	useful energy
$v$	volumetric

CO<sub>2</sub> (R744) is the natural refrigerant which imposes negligible impact on the environment. The heat transport properties of CO<sub>2</sub> are favorable especially near to its critical point compared to other conventionally used working fluids. Although CO<sub>2</sub> has a lower critical point, the use of such refrigerant in the evacuated tube U-pipe solar collector has recently been researched in the both transcritical and supercritical cycles [3, 4]. A unique property of CO<sub>2</sub> over other refrigerants is that it experiences only a very small change in saturation temperature with a given change in saturation pressure. Thus the efficiency of CO<sub>2</sub> direct expansion heat pump system does not affect much with the large change in pressure [5]. Figure 1 shows a schematic diagram of the system used in the present study that comprises of two main circulation loops: CO<sub>2</sub> loop and water loop. In this paper, CO<sub>2</sub> is proposed as a working fluid to study the performance of DX-SAHP water heater in a transcritical cycle. To validate the developed model, the simulated results have been compared with the experimental data reported in the literature. Details of the developed model are discussed in the following section.

## 2. Mathematical Modeling

A transcritical CO<sub>2</sub> heat pump cycle has been modeled by utilizing U-pipe evacuated tube solar collector as an evaporator. The thermal performance of the system is predicted based on the following general assumptions: (a) steady state heat transfer process while in operation; (b) compression process is considered as adiabatic but not isentropic; (c) pressure drop in connecting pipes are negligible; (d) isenthalpic expansion process is considered. Above assumptions are taken into consideration for describing the governing equations of the various components of the heat pump system. The

system comprises of four basic system components, such as evaporator/solar collector, compressor, gas cooler/heat exchanger, and expansion valve.

### 2.1 Solar collector/evaporator model:

U-pipe evacuated tube solar collector has been used for modeling the evaporator component. The useful energy gain by the solar collector is given by,

$$Q_u = A_{coll} F' [I(\tau_g \alpha_g) - U_L(T_f - T_a)] \quad (1)$$

where the overall heat loss coefficient  $U_L$  is defined as

$$U_L = U_t + U_e \quad (2)$$

Edge loss coefficient,  $U_e$  is assumed to be negligible but the top loss coefficient,  $U_t$  is a function of both convective and radiative heat losses [6]. The collector efficiency factor,  $F'$  can be expressed by Hottel–Whilliar Bliss equation as:

$$F' = \frac{\frac{1}{U_L}}{W \left[ \frac{1 + \frac{U_L}{C_b}}{U_L [d + (W - d)F]} + \frac{1}{C_b} + \frac{1}{h_f \pi d} \right]} \quad (3)$$

The standard fin efficiency,  $F$  can be expressed as

$$F = \frac{\tanh \left[ \frac{m(W - d)}{2} \right]}{m(W - d)/2} \quad (4)$$

where  $m$  is defined as

$$m = \left[ \frac{U_L}{\lambda \delta \left( 1 + \frac{U_L}{C_b} \right)} \right]^{\frac{1}{2}} \quad (5)$$

The pressure drop inside the collector tubes and heat transfer coefficients are determined based on generalized equations reported in the literature [4, 7–9]. As a transcritical cycle deals with both the two-phase and the supercritical vapor zone, the working fluid CO<sub>2</sub> enters the collector tubes as two-phase mixtures and leaves the collector outlet as superheated vapor. The heat transfer coefficient in two-phase is determined using the correlation of Jung et al. [7] and pressure drop is predicted by Lockhart-Martinelli equation [8]. In the single phase vapor flow heat transfer coefficient is calculated using the Gnielinski relation [9] and pressure drop is determined by the usual Darcy-Weisback equation. The temperature of the CO<sub>2</sub> fluid inside the U-tube is determined by Zhang et al. [4] using a finite volume approach.

The thermal resistance in the selectively coated, U-pipe, and fin is considered to be negligible and hence it assumed to be at the same temperature. The convection coefficient of air can be predicted based on the wind velocity.

$$h'_a = 5.7 + 3.8V \quad (6)$$

where  $V$  is the wind velocity in m/s.

The efficiency of the collector is expressed as

$$\eta_{coll} = \frac{Q_u}{A_c I} \quad (7)$$

The energy absorbed by the refrigerant can also be described by the enthalpy changes between the inlet and outlet of the collector and the mass flow rate.

$$Q_{\#} = \dot{m}_r (h_1 - h_4) \quad (8)$$

## 2.2 Compressor model:

Mass flow rate of the refrigerant through the compressor is expressed as

$$\dot{m}_r = \rho_{suc} \eta_v V_s \frac{N}{60} \quad (9)$$

where compressor speed  $N$  is measured in rpm. The volumetric efficiency,  $\eta_v$  of the semi-hermetic compressor can be determined [10] as

$$\eta_v = 0.9207 - 0.0756 \left( \frac{P_{dis}}{P_{suc}} \right) + 0.0018 \left( \frac{P_{dis}}{P_{suc}} \right)^2 \quad (10)$$

The isentropic efficiency of the compressor can be expressed by eqn. (11) and calculated by employing pressure ratios of semi-hermetic compressor of eqn. (12) [10]

$$\eta_s = \frac{h_{2s} - h_1}{h_2 - h_1} \quad (11)$$

$$\eta_s = -0.26 + 0.7952 \left( \frac{P_{dis}}{P_{suc}} \right) - 0.2803 \left( \frac{P_{dis}}{P_{suc}} \right)^2 + 0.0414 \left( \frac{P_{dis}}{P_{suc}} \right)^3 - 0.0022 \left( \frac{P_{dis}}{P_{suc}} \right)^4 \quad (12)$$

and finally the compressor work can be calculated as

$$W_{com} = \dot{m}_r \frac{(h_{2s} - h_1)}{\eta_s} \quad (13)$$

## 2.3 Gas cooler/Heat exchanger model:

The heat exchanger modeled in this system is considered to have concentric-tubes with refrigerant flowing through the inner tube and water flowing through the annular space between tubes in counter flow direction. Assuming negligible heat losses to the ambient, the energy balance for the heat exchanger based on 1<sup>st</sup> law of thermodynamics can be written as

$$\dot{Q} = \dot{m}_r (h_{r,i} - h_{r,o}) = \dot{m}_w (h_{w,o} - h_{w,i}) \quad (14)$$

$$\dot{Q} = \dot{m}_r C_{p,r} (T_{r,i} - T_{r,o}) = \dot{m}_w C_{p,w} (T_{w,o} - T_{w,i}) \quad (15)$$

The heat transfer rate can also be determined from the effectiveness-NTU method for the counter-flow heat exchanger as

$$\dot{Q} = \epsilon C_{\min}(T_{r,i} - T_{w,i}) \quad (16)$$

where effectiveness of counter-flow is

$$\epsilon = \frac{1 - e^{-NTU(1-R)}}{1 - R e^{-NTU(1-R)}} \quad (17)$$

The number of transfer units is defined as

$$NTU = \frac{UA}{C_{\min}} \quad (18)$$

where the overall heat transfer conductance  $UA$  is defined as

$$UA = \left( \frac{1}{h'_r A_c} + \frac{\ln(d_o/d_i)}{2\pi(\Delta L)k_{wall}} + \frac{1}{h'_w A_w} \right)^{-1} \quad (19)$$

The water side heat transfer coefficient is determined by Nusselt relationship given by Petukhov and Roizen [11] and the supercritical CO<sub>2</sub> heat transfer coefficient is determined by using Pitla et al. [12].

The delivered heat energy to the water can also be defined as a function of enthalpy relation:

$$Q_H = \dot{m}_r (h_4 - h_2) \quad (20)$$

## 2.4 System performance:

The COP of the heat pump system is expressed as

$$COP = \frac{\text{Heating Capacity}}{\text{Total energy input}} = \frac{Q_H}{W_c} \quad (21)$$

## 3. System simulation

The thermodynamic transcritical cycle of the proposed heat pump system was determined by the heat transfer processes associated with the CO<sub>2</sub> flow through the solar collector and the compression process by the compressor. The working fluid was assumed to be uniformly distributed in the U-pipe of the collector tubes. In order to evaluate the transcritical cycle performance of using CO<sub>2</sub> in the solar-assisted heat pump system, a constant compressor input is considered for the each simulation cycle and henceforth determined the supercritical nature of CO<sub>2</sub> in the the following components in a steady state operating conditions. The simulation procedure as continued until the enthalpy change across the expansion valve was minimized. It should be pointed out that CO<sub>2</sub>

has a comparatively low critical point ( $P_c = 7.38$  Mpa,  $T_c = 31.1^\circ\text{C}$ ). Supercritical  $\text{CO}_2$  near to its critical point shows rapid variations in thermodynamic and transport properties with a little change in temperature (near the pseudo-critical point). This could be effectively explained to harness useful energy in the evaporator section.

This model evaluates the performance of solar-assisted heat pump cycle for water heating purposes when exposed to work during winter and spring conditions pertaining to North Dakota region, USA. All the meteorological data used in the model are from Ref. [13]. The performance of the transcritical cycle, such as COP, heat delivery rate, collector efficiency, and mass flow rate of  $\text{CO}_2$  is simulated for various compressor speeds ranging from 900 – 1500 rpm. The base parameters used in the simulation model are listed in Table 1. Thermophysical fluid properties, such as enthalpy, entropy, specific heat, and density are determined by using REFPROP 8.0 software package.

A counter-flow heat exchanger of 15m length with inner tube having internal and external diameter of 5 and 7 mm, respectively, and outer tube internal diameter of 16mm has been modelled in this study. The flow rate and inlet water temperature were assumed to be constant at 0.05 kg/s and  $7^\circ\text{C}$ , respectively [15].

Table 1. Simulation parameters considered in this work [4, 14]

Component Size	Parameters	Values
Solar collector:	Evacuated-tube U-pipe	
Outer glass tube	Outer diameter	47 mm
	Thickness	1.2 mm
	Thermal conductivity	1.25 W/(mK)
	Solar transmittance	0.90
	Solar absorbance	0.05
	Thermal emittance	0.83
Absorber tube	Outer diameter	37 mm
	Thickness	1.2 mm
U-tube	Outer diameter	8 mm
	Thermal conductivity	400 W/(mK)
Collector tube length		1700 mm
Bond conductance		30 W/(mK)
Compressor:	Reciprocating-type, hermetic compressor	
Displacement volume		0.00001972 $\text{m}^3$

## 4. Results and Discussions

In order to avoid two-phase properties at the inlet, compressor was assumed to deal only with the vapor phase to the discharge state of either vapor or supercritical state. Thus the solar collector will undergo a part of heating process in the two-phase region and a part of it in the vapor region before the refrigerant entering into the compressor inlet. A  $\text{CO}_2$   $P$ - $h$  diagram is shown in Fig. 2 considering a typical spring weather day of Fargo, ND [16]. It is seen from the simulated state that a transcritical operating condition is achieved throughout a steady state operating cycle. It is found that a heat sink of relatively low-temperature ( $-5^\circ\text{C} - 10^\circ\text{C}$ ) plays an important role to ensure efficient transcritical operation.

Figure 3 shows the validation in collector efficiency the ratio of the temperature difference of fluid and ambient to the solar intensity. The average efficiency (52%) was compared with the experimental value [3] and was found to agree well within an error difference of 13%. The predicted efficiency values are higher than water driven solar collector. The performance of the heat pump system is evaluated in terms of COP values and the heat extraction rate at the heat exchanger for the solar intensity varying from 550 to 700  $\text{W}/\text{m}^2$  with a compressor speed ranged from 900 to 1500 rpm. Figure 4 shows the variation in the system COP as well as rate of heat output with change in compressor speed. Comparatively lower compressor speeds, 1000 to 1100 rpm, the system could attain a COP around 2 – 2.5 with a heat extraction rate of 2.4 – 3.4 kW. However, when the compressor speeds were increased beyond 1100 rpm, COP and heat output reduced since, required work input was much higher than the effective raise in the working fluid temperature.

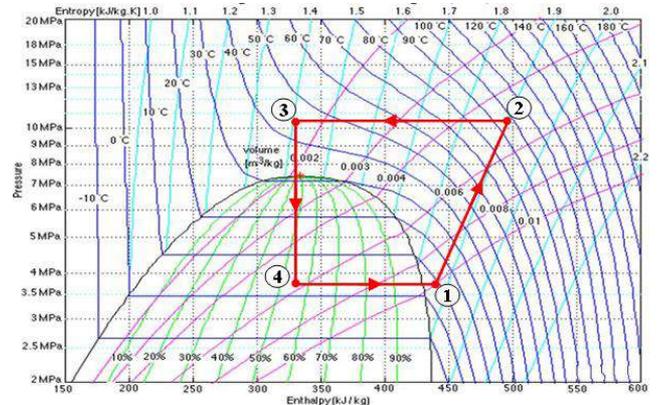


Fig. 2:  $P$ - $h$  diagram of  $\text{CO}_2$  in a steady-state cycle of simulated model.

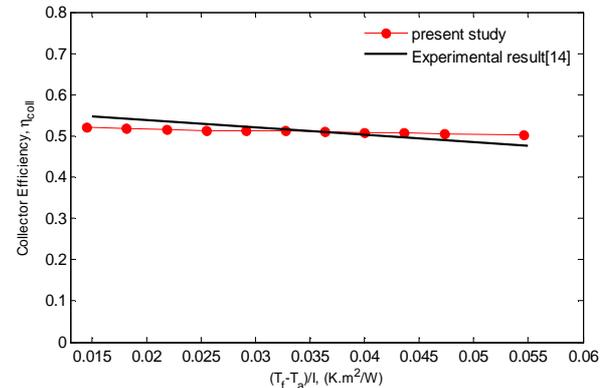


Fig. 3: Evacuated tube solar collector efficiency curve at a solar radiation of  $I = 550 \text{ W}/\text{m}^2$ .

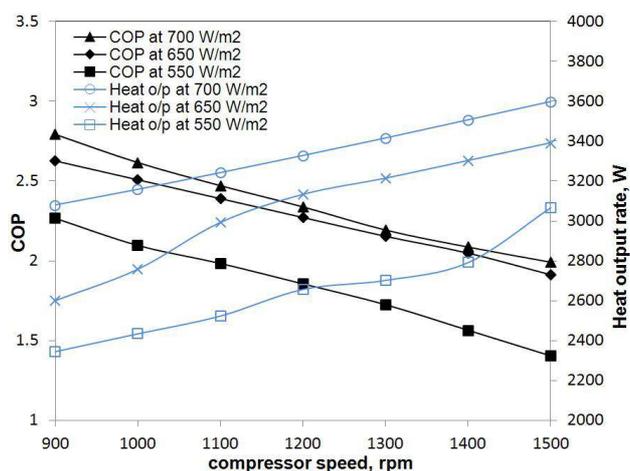


Fig. 4: Variation of system COP and heat delivery rate with compressor speed.

## 5. Conclusion

Thermal performance of SA-DXHP water heater by using CO<sub>2</sub> as a refrigerant for the moderate weather conditions of North Dakota has been simulated and is discussed in this paper. The results were then optimized for varying compressor speed. Results revealed that by decreasing compressor speed from 1500 to 900, the COP can be improved by an average of 57%. The efficiencies of the solar collector were also predicted (50–55%) and found a good agreement with the experimental values. Even though U-pipe assisted evacuated tube solar heat pump systems pertain higher initial cost, the environmentally friendly and economically cheap CO<sub>2</sub> have better prospect in solar thermal applications.

## Acknowledgement

This research was in part supported by a grant from the Pakistan-US Science and Technology Cooperation Program, US Department of State (jointly administered by the National Academics and Higher Education Commission of Pakistan).

## References

[1] S.K. Chaturvedi and J.Y. Shen, Thermal performance of a direct expansion solar-assisted heat pump, *Solar Energy* 1984; 33(2): 155-162.

- [2] Y. Kim and T. Seo, Thermal performances comparisons of the glass evacuated tube solar collectors with shapes of absorber tube, *Renewable Energy* 2007; 32(5): 772-7795.
- [3] X.R. Zhang and H. Yamaguchi, An experimental study on evacuated tube solar collector using supercritical CO<sub>2</sub>, *Applied Thermal Engineering* 2008; 28: 1225-1233.
- [4] X.R. Zhang, H. Yamaguchi, D. Uneno, K. Fujima, M. Enomoto and N. Sawada, Analysis of a novel solar energy powered Rankine cycle for combined power and heat generation using supercritical carbon dioxide, *Renewable Energy* 2006; 31: 1839-1854.
- [5] M.H. Kim, J. Pettersen and C.W. Bullard, Fundamental process and system design issues in CO<sub>2</sub> vapor compression systems, *Progress in Energy and Combustion Science* 2004; 30(2): 119-174.
- [6] J.A. Duffie and W.A. Beckman, *Solar Engineering of Thermal Processes*, 1980; 28-143.
- [7] D.S. Jung and R. Radermacher, Prediction of evaporation heat transfer coefficient and pressure drop of refrigerant mixtures in horizontal tubes, *International Journal of Refrigeration* 1993; 16(3): 201-209.
- [8] R.W. Lockhart and R.C. Martinelli, Proposed correlation of data for isothermal two-phase two component flow in pipes, *Chemical Engineering Progress* 1949; 45(1): 39-48.
- [9] V. Gnielinski, New equations for heat and mass transfer in turbulent pipe and channel flow, *International Journal of Chemical Engineering* 1976; 16: 359-368.
- [10] T.M. Ortiz, D. Li and E.A. Groll, Evaluation of the performance potential of CO<sub>2</sub> as a refrigerant in air-to-air air conditioners and heat pumps: System modeling and analysis. 2003; Final Report, ARTI.
- [11] B.S. Petukhov, B.S. Roizen and L.I. Lewis, Generalized relationships for heat transfer in a turbulent flow of a gas in tubes of annular section, *High temperature-USSR* 2, 1964: 65-68.
- [12] S.S. Pitla, E.A. Groll and S. Ramadhyani, New correlation to predict the heat transfer coefficient during in-tube cooling of turbulent supercritical CO<sub>2</sub>, *International Journal of Refrigeration* 2002; 25(7): 887-895.
- [13] North Dakota Agricultural weather network (NDAWN Center), 2000-2011 North Dakota State University. Accessed online: <http://ndawn.ndsu.nodak.edu/daily-table-form.html>
- [14] L. Ma, Z. Lu, J. Zhang and R. Liang, Thermal performance analysis of the glass evacuated tube solar collector with U-tube, *Building and Environment* 2010; 45(9): 1959-1967.
- [15] North Dakota State Climate Office. North Dakota Annual Average Temperature. Accessed online: <http://www.ndsu.edu/ndsco/temp/monthly/2010.html>
- [16] E.W. Lemmon, M.L. Huber and M.O. McLinden, NIST Standard Reference Database 23, Reference Fluid Thermodynamic and Transport Properties (REFPROP), Version 8.0, National Institute of Standards and Technology 2007.